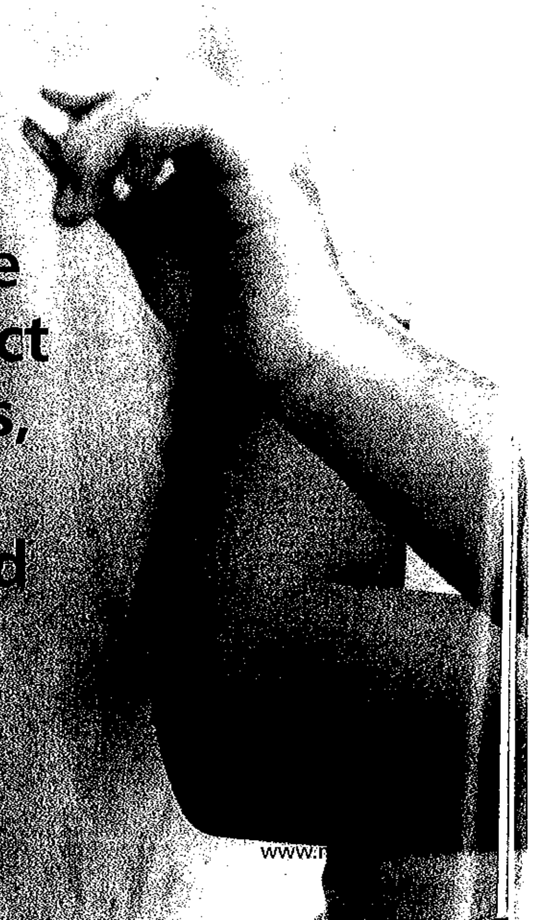
in the Early Elementary Years



To help children understand the concrete concept that an abstract orthographic symbol represents, let's apply the same strategies we use for teaching background knowledge in reading.

By Chris R. Cain and Valerie N. Faulkner

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The widely adopted Common Core State Standards for Mathematics (CCSSI 2010) are designed to deepen instruction of number sense and will demand that elementary school teachers have a strong understanding of number. These changes arrive at a time when it is still understood that teachers and the curriculum in the United States have not been fundamentally driven by number sense connections (Ball and Cohen 1996). Teachers, therefore, are faced with the need to reflect on their own instructional choices and to make changes in their classrooms—changes that encourage the development of number sense in their students in keeping with the demands of the Common Core State Standards for Mathematics (CCSSM) and that go beyond what they have formerly thought about number (Ball and Cohen 1996). In our professional development with teachers from across our southeastern state, we have found that providing a model to develop the teacher's own sense of number is crucial. This model (see **fig. 1**) offers teachers an opportunity to reflect on their lessons and consider whether they have made mathematics connections that develop number sense in their students. By consciously exploring their own sense of number, teachers take an important step toward deepening their instruction in line with the CCSSM and creating classrooms that develop students' ability to reason abstractly and quantitatively, model situations with mathematics, and make use of mathematical structures.

Consistent with what we know about the importance of planning and reflection in lesson study (Hiebert and Stigler 2000; Stigler and Hiebert 2004) and the importance of

FIGURE 1

The Components of Number Sense

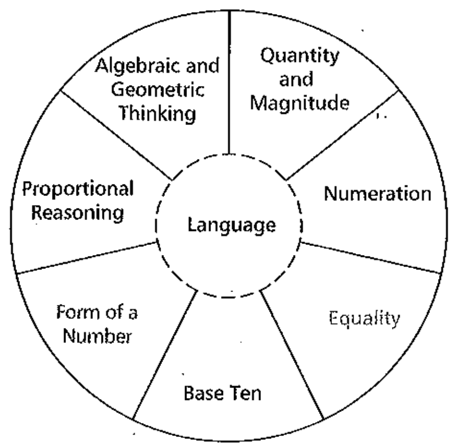

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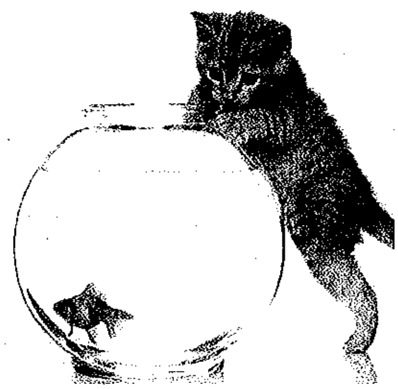
FIGURE 2

What is three?

A numeral 3 is not three, just as *c-a-t* is not a cat.



This is not a cat either. It is, clearly, just a *picture* of a cat. You might say it illustrates the *curious* nature of a cat.

And so it is for ***.
*** is not three, it is a *picture* of three. It represents the *quantity* nature of three.

teacher content knowledge (Ball, Thames, and Phelps 2007; Ball et al. 2008), we have found that working with teachers to improve their instructional practice begins with activities and discussions that enable them to relearn and reconceive what it means to understand and teach mathematics (Hiebert and Stigler 2000; Ball, Thames, and Phelps 2007).

For teachers of young students, the critical components of number sense are quantity, magnitude, numeration, different forms of a number, equality, and the language we use to describe them. The concepts we discuss in this article connect to the curriculum and process goals of the CCSSM and also tie directly to the Number and Operations Focal Points (NCTM 2006) for prekindergartners (who should understand that number words refer to quantity) and for kindergartners (who should use numbers, including written numerals, to represent quantities).

Teaching number

In language arts, we intuitively understand that the word *c-a-t* is not a cat. It is silly to even state something so obvious. Our deep understanding of the objects and actions around us make this discussion unnecessary. Once students learn to *decode* words (correctly analyze and interpret the spoken or graphic symbols of a language) and *attack* words (convert graphic symbols into language), they have the implicit understanding that the words never are the things they describe.

Now consider number. Take the numeral 3. Just as *c-a-t* is not a cat, the numeral 3 is not the quantity three. *T-h-r-e-e* is not three either. Three dots, . . . , are a little closer to the reality, just as a picture of a cat (see **fig. 2**) is a bit closer to a real cat. But if the numeral 3, the word *three* and . . . are not three, what *is* three?

Again, consider the analogy to reading. How do we develop an understanding of words in our students? First, of course, we teach students to decode. But we do not mistake decoding for comprehension. We know that some students are "word callers" who can "read" but then again not really *read*. And we also understand that some students may not have experienced the meanings behind certain words. In these cases, we work to bring understanding of the actual object or action to our students to build their vocabulary. For instance, a student reading about koala bears might be provided with pictures of koala bears; and koala bears will be compared and contrasted with known animals to get a sense of the size, feel, and nature of a koala bear. We will make sure that students understand where koala bears live and how common or rare they are. These are all basic

"I'm not a Tar Heel; I'm four!"

LAURA EISENBERG/ISTOCKPHOTO.COM

We must adapt our instructional practices as we better understand how children learn to make sense of number.

and one. Just as in reading, wherein some children develop the skill of word calling, in mathematics they develop the skill of number calling or fact memorization (without understanding). Practice with subitizing is a fun early step in ensuring that our students are not number calling but understanding what is underneath the numeral. This connects most directly to the number sense components of quantity and different forms of number.

As these same young children are then asked to compose objects, they are often given mathematical statements, such as $3 + 4 = \_\_\_$. At this point, the teacher might say little more than, "If I put three and four together, I will get seven." Many children will memorize these facts without understanding them, and others will struggle to see how we put the symbol 3 and the symbol 4 together and have them "make" the symbol 7; many children see a 3 and a 4 together as 34. Some of these difficulties that students have in understanding number are developmental, but others may stem from the way number is handled with very young students. Here is an example of this misconception in action: A teacher puts paper, markers, and ten objects in front of each student. The teacher asks that each child put five in their hand. The teacher looks around the room to confirm that every student has five objects left on his or her desk. She discovers that one child still has ten manipulatives on his desk. She asks the student, "Why didn't you pick up five?"

The young man replies, "You just said, 'Put five in your hand,'" as he opens his hand to reveal the number 5 written in green marker. Nonetheless, this same child had memorized his fives facts without being able to explain what was really happening.

Consider an older student who likewise takes these abstract symbols and processes them as though they are concrete objects (see **fig. 4**).

The tendency to think of number identification as a skill that can be taught without context is ingrained in our current instruction. In a recent class observation, we saw students who had been identified by the teacher as struggling students. And indeed, these kindergartners could not order blocks from one to ten (one block, two blocks, three blocks, etc.) in the late spring—a key objective in the CCSSM. But the teacher, who knows that these students must be able to "read" numbers up to thirty when kindergarten ends, spends her instructional time with

FIGURE 4

This example of a student's work confounds number symbols with concrete objects:

8 – 5 = 8   7 – 4 = 7

Student explanation: "Eight take away five is eight. Seven take away four is seven."

This student means, literally, that if you have an eight and a four and you take away the four, you still have the eight. She is applying concrete qualities to the number symbols. She does not see eight as composed of other numbers; she sees eight as a *thing* in and of itself.

Referring back to our understanding of words and reading, the student is essentially saying, "If I have a cat and a dog, and I take away the dog, I still have the cat." Here she *has* an 8 and a 5, and she takes away the 5, so she still has the 8. Her reading of the symbols is merely number calling without comprehension of the numbers themselves.

FIGURE 5

This example of dialogue between a teacher and her struggling students de-emphasizes the comprehension of number.

Point to number 28 on this table of numbers. [*The children do so or are redirected to point to the correct number.*] **Who can point to the number 24? Who can point to the number 23?**

I know which one that it is; it's the 2 and the 3.

[*The teacher agrees with the student and reinforces this digit-based conception of the number.*]

24	31	28
28	25	26
30	27	29

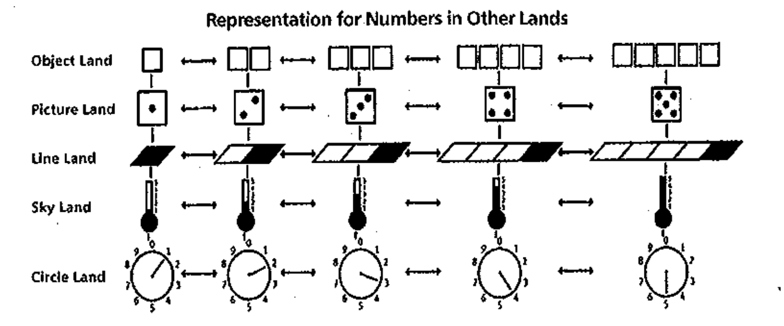
these students, working on that objective. This is a teacher who works hard and does an excellent job with her students in virtually all other areas of the curriculum. Yet her mathematics instruction for these students involves having them look at cards of numbers in the twenties and thirties (placed out of sequence) and pointing as she asks which number is twenty-eight, which number is twenty-four, and so on (see **fig. 5**). She is emphasizing rote and symbolic numeration skills at the expense of the many other components of number. This approach does not demonstrate to students the *structure* of how numbers are related to one another, nor does it support their ability to *reason abstractly and quantitatively*. Without these skills, students are strictly memorizing number names, not understanding the quantities represented in abstract form. These students are learning to "call" these larger numbers before they understand what even smaller numbers represent and how they are connected to the number line and *magnitude*. Students are often asked to "read" large numbers before they understand smaller ones. This is problematic because it engenders misconceptions about the very nature of number symbols and number itself.

Often children have little conceptual understanding that combining two symbols means that they are being asked to add (put together) two sets of objects: 3 (***) + 4 (****) = 7 (*** + ****). Even if teachers draw this representation on the board, children have often developed a false understanding of these concepts. We believe that teachers default to this type of instruction because the deeper connections were not taught to them when they were young, and curricula in the past have asked teachers to cover so many topics that they do not have time to reflect on these deeper number sense connections.

Developing comprehension

So what would it look like to teach early number comprehension using the Components of Number Sense and addressing the CCSSM? To teach any number with meaning, we must develop students' sense of quantity and magnitude as well as the different forms that equal numbers can take. For young students, this means developing the preskills of verbal counting, one-to-one correspondence, subitizing, and counting within a set for cardinality (Clements and Sarama 2009). A teacher may put "3" on the board, show three manipulatives to the students, and discuss with them that this symbol represents the number of objects in this set. However, once a teacher has time to reflect, she may begin to take this even further and build contextual understanding into

FIGURE 6



her instruction of number. The Components of Number Sense might guide her thinking by encouraging her to consider if she has continued with these important pre-skills to deeply teach quantity, different forms of number, equality, and numeration. Strong instruction would connect as many of these components as possible, even with such a small number as three or five. Students must have an opportunity to count a set of manipulatives with one-to-one correspondence to the cardinal amount of 3. They will need time to "see" three through activities that involve not just counting by ones, but also subitizing. To make certain that students fully understand how three is composed, the teacher might demonstrate how three can be composed of three objects in one hand and no objects in the other, two objects in one hand and one in the other, and so on.

Numbers in context

Strong instruction of number will also include work with magnitude and developmentally appropriate explorations of the number line. Such explorations allow students to relate symbolic numbers to quantities and understand number as it relates to other numbers (Griffin and Case 1997; Moss and Case 1999; Griffin 2002). Again, the use of the number line provides a tangible support for not only the student but also the teacher and the types of questions and activities the number line will engender. Multiple concrete ways exist for introducing number with meaning, such as counting the number of—

- **desks** in the room;
- **boys** versus girls in the class and placing each quantity on the number line; and
- **steps** from one fixed point to another and then using number to compare quantities.

With letters and words, students must understand that context is crucial to interpretation and meaning. With numbers, each symbol has a constant meaning, but that meaning must be understood deeply and in context. In reading, we develop context for students through experiences, practice, and text analysis; in early math development, the number line is the context. By playing games with the number line, and by continually using the number line to make sense of number, students begin to know number, not just read numbers. Students do not know three unless they can show or discuss with you that three is one larger than two and one smaller than four.

Students know five when they can relate it to other numbers, such as four or eight (Griffin 2002; Griffin 2005). These relationships are best built both by developing a sense of quantity *and* a sense of magnitude (see **fig.** 6). Just as my comprehension for a Koala bear deepens when I know how big it is and where it lives in the world, so does my comprehension for three deepen when I know how large it is and where it "lives" on the number line. Finally, playing with the number line also allows students to develop clear meanings for important early math vocabulary such as *before, after, more than, less than*, and *equal to*. These meanings are critical to addressing the goals built into the CCSSM.

We cannot underestimate a young student's penchant for making the symbolic concrete and for misunderstanding number. While at a recent sporting event, one of the authors overheard a young child respond vehemently to someone asking if this child were a Tar Heel: "I'm not a Tar Heel; I'm four!" Let this remind us that children are prone to misconceptions about the nature of number and that our work with young children must involve development of their initial number comprehension in much the same manner we develop comprehension in any other topic. By remembering that "this is not three," teachers are positioned to reflect on the components of number sense, to emphasize teaching students to read mathematical symbols with deep contextual meaning, and to develop instructional habits that are compatible with the goals of the Common Core State Standards for Mathematics.

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